Spherical cap discrepancy of perturbed lattices under the Lambert projection

Damir Ferizović KU Leuven damir.ferizovic@kuleuven.be

Given any full rank lattice $\Lambda \subset \mathbb{R}^2$ and a natural number N, we regard the point set $\Lambda/N \cap (0,1)^2$ under the Lambert map to the unit sphere \mathbb{S}^2 , and show that its spherical cap discrepancy is at most of order N, with leading coefficient given explicitly and depending on Λ only. The proof is established using a lemma that bounds the amount of intersections of certain curves with fundamental domains that tile \mathbb{R}^2 , and even allows for local perturbations of Λ without affecting the bound, proving to be stable for numerical applications. A special case yields the smallest constant for the leading term of the cap discrepancy for deterministic algorithms up to date.

References

[1] D. Ferizović. Spherical cap discrepancy of perturbed lattices under the Lambert projection. In Preparation, 2022.