Adaptive meshfree solving of linear PDEs: Analysis of target-data dependent greedy kernel methods

Tizian Wenzel
Institute for Applied Analysis and Numerical Simulation,
University of Stuttgart, Germany
tizian.wenzel@mathematik.uni-stuttgart.de

We consider meshless solving of PDEs

$$Lu = f \quad \text{on } \Omega, \quad u = g \quad \text{on } \partial\Omega \tag{1}$$

via symmetric kernel collocation by using greedy kernel methods. In this way we avoid the need for a mesh generation, which can be challenging for non-standard domains Ω or manifolds. We introduce and discuss different kind of greedy selection criteria, such as the PDE-P-greedy and the PDE-f-greedy.
Subsequently we analyze the convergence rates of these algorithms and provide bounds on the approximation error in terms of the number of greedily selected points. Especially we prove that target-data dependent algorithms exhibit faster convergence rates.
The provided analysis is applicable to PDEs both on domains and manifolds. This and the advantages of target-data dependent algorithms is highlighted by numerical examples.

Joint work with: Daniel Winkle, Gabriele Santin, Bernard Haasdonk.

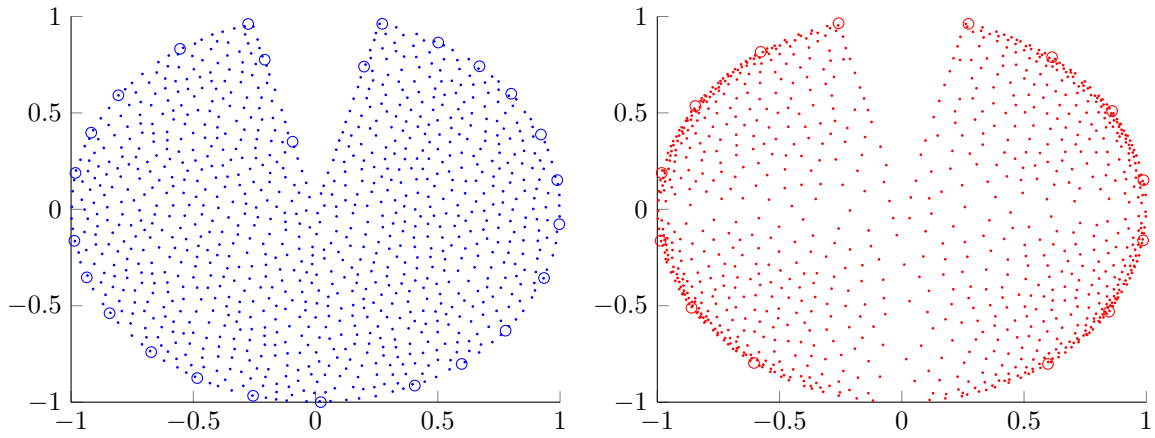

Figure 1: Visualization of the selected points for PDE-P-greedy (left) and PDE-f-greedy (right) for a Laplace equation with Dirichlet boundary conditions on a domain which is given by a circle without a cone. The circled points correspond to points on the boundary $\partial\Omega$. The PDE-f-greedy points (right) are better adapted to the given PDE and the domain and therefore give a better approximation of the solution.

References

[1] Tizian Wenzel, Daniel Winkle, Gabriele Santin, Bernard Haasdonk Adaptive meshfree solving of linear PDEs: Analysis of target-data dependent greedy kernel methods. *In preparation*, 2022.

[2] Tizian Wenzel, Gabriele Santin, Bernard Haasdonk Analysis of target data-dependent greedy kernel algorithms: Convergence rates for f-, $f \cdot P$- and f/P-greedy. *ArXiv*, (2105.07411), 2021. Submitted.