Optimal spline subspaces for outlier-free isogeometric analysis

Hendrik Speleers
University of Rome Tor Vergata
speleers@mat.uniroma2.it

Spectral analysis can be used to study the error in each eigenvalue and eigenfunction of a numerical discretization of an eigenvalue problem. For a large class of boundary and initial-value problems the total discretization error on a given mesh can be recovered from its spectral error. This is of primary interest in engineering applications.

The isogeometric approach for eigenvalue problems has been widely investigated in the literature; see, e.g., [1, 3, 4]. Maximally smooth spline spaces on uniform grids are an excellent choice for addressing eigenvalue problems. Yet, they still present a flaw: a very small portion of the eigenvalues are poorly approximated and the corresponding computed values are much larger than the exact ones. These spurious values are usually referred to as outliers. The number of outliers increases with the degree p. However, for fixed p, it is independent of the degrees of freedom for univariate problems, while a "thin layer" of outliers is observed in the multivariate setting.

Outlier-free discretizations are appealing, not only for their superior description of the spectrum of the continuous operator, but also for their beneficial effects in various contexts, such as an efficient selection of time-steps in (explicit) dynamics and robust treatment of wave propagation. For a fixed degree, the challenge is to remove outliers without loss of accuracy in the approximation of all eigenfunctions.

In this talk we discuss isogeometric Galerkin discretizations of eigenvalue problems related to the Laplace operator subject to any standard type of homogeneous boundary conditions conditions in certain optimal spline subspaces [5]. Roughly speaking, these optimal subspaces are obtained from the full spline space defined on specific uniform knot sequences by imposing specific additional boundary conditions. The spline subspaces of interest have been introduced in the literature some years ago when proving their optimality with respect to Kolmogorov n-widths [2]. For a fixed number of degrees of freedom, all the eigenfunctions and the corresponding eigenvalues are well approximated, without loss of accuracy in the whole spectrum when compared to the full spline space. Moreover, there are no spurious values in the approximated spectrum. In other words, the considered subspaces provide accurate outlier-free discretizations in the univariate and in the multivariate tensor-product case. The role of such spaces as accurate discretization spaces for addressing general problems is discussed as well.

Joint work with: Carla Manni, Espen Sande.

References

[1] J.A. Cottrell, A. Reali, Y. Bazilevs, and T.J.R. Hughes. Isogeometric analysis of structural vibrations. *Computer Methods in Applied Mechanics and Engineering*, 195:5257–5296, 2006.

[2] M.S. Floater and E. Sande. Optimal spline spaces for L^2 n-width problems with boundary conditions. *Constructive Approximation*, 50:1–18, 2019.

[3] C. Garoni, H. Speleers, S-E. Ekström, A. Reali, S. Serra-Capizzano, and T.J.R. Hughes. Symbol-based analysis of finite element and isogeometric B-spline discretizations of eigenvalue problems: Exposition and review. *Archives of Computational Methods in Engineering*, 26:1639–1690, 2019.

[4] R.R. Hiemstra, T.J.R. Hughes, A. Reali, and D. Schillinger. Removal of spurious outlier frequencies and modes from isogeometric discretizations of second- and fourth-order problems in one, two, and three dimensions. *Computer Methods in Applied Mechanics and Engineering*, 387:114115, 2021.

[5] C. Manni, E. Sande, and H. Speleers. Application of optimal spline subspaces for the removal of spurious outliers in isogeometric discretizations. *Computer Methods in Applied Mechanics and Engineering*, 389:114260, 2022.