Sampling recovery in L_2

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We consider the problem of recovering a function $f\colon\Omega \to \mathbf{C}$ belonging to some class F based on a finite number of samples. The class F reflects our a priori knowledge about the function. Here, Ω is any compact domain or manifold and F is any compact subset of $C(\Omega)$. The error is measured in a worst case scenario (over the function class) and with respect to the L_2-distance. The following general result was recently obtained by the authors:

If the Kolmogorov widths of F show a polynomial decay of order $s > 1/2$,
then there is a weighted least squares estimator that achieves the same rate of convergence.

We discuss this result and address the following questions: What does the algorithm look like? What can be said in the case $s \leq 1/2$? What results do we obtain for the tractability of the problem in high dimensional settings? We also relate to recent results of Nagel/Schäfer/Ullrich, Temlyakov and Cohen/Dolbeault.

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References

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