Recognition and approximation of space curves on 3D digital models

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Space curves play a fundamental role in conveying an object shape. Contours and curvilinear profiles are significant in manufacturing, art, design and medical applications. For instance, in reverse engineering, 3D scanning devices are used to digitize and validate a manually optimized physical prototype and it is extremely important that no details of the scanned object, e.g. sharp edges, corners and in general feature curves, are lost during the acquisition process. Indeed, when 3D models are acquired by scanning real objects, the resulting geometry does not explicitly encode these curves, especially when it is affected by noise or missing parts. This behavior is mainly due to measurement uncertainty, sampling resolution, or occlusion during the acquisition whereas, in applications like the digitisation of archaeological artefacts, these objects might be damaged, and then curves are partially missing.

The problem of recognising space curves and providing a mathematical representation of them can be addressed through the use of the Hough transform technique, which is well known for recognising curves in the plane and surfaces in space [1], but not yet sufficiently explored for space curves. Such a technique is robust to noise and outliers and does not suffer from missing parts. In this work, we will present and analyse the Hough transform (HT) to recognise and approximate space curves in 3D digital models (see [2]), a problem that is not currently addressed by the standard HT. In our approach, we take advantage of a recent HT formulation for algebraic curves to define both parametric and implicit curve representations. Specifically, we extend a first approach that applies directly the HT to the recognition of curves in space in parametric form, but it is limited to the case where the number of parameters is equal to the number of equations [3]. Regarding the implicit representation, we implement and follow the strategy defined in [4] that exploits the evaluation of the implicit functions and compares them with two theoretical bounds. To apply the HT to space curves, we assume that: *i*) the family of curves in which to search for the solution holds a regular parameterization for curves expressed in a parametric form; *ii*) the equations that define these curves are analytical for curves in implicit form.

The limited availability of templates for space curves has probably reduced the interest to the space curve fitting problem. To overcome this issue, we extend the existing dictionary of space curves by defining two types of families: we call *type I curves* the families of space curves equipped with an explicit representation, whereas we label *type II curves* the families obtained as the intersection of a quadric surface and a cylinder having a plane curve as its directrix, exploiting the large variety of plane curves available [5].

Joint work with: Silvia Biasotti, Bianca Falcidieno.

References

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