Aesthetic planar curves

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In Geometric Design, it is of interest the representation of curves and surfaces that are aesthetically pleasing. In order to have a notion amenable to implementation in CAGD, *aesthetic curves* have been defined as those with monotonic curvature and, for spatial curves, monotonic torsion. There are several approaches to obtain aesthetic Bézier curves, but we will follow the lead of [4] and [3].

In [4], Mineur, Lichah, Castelain and Giaume obtain the edges of the control polygon of a planar Bézier spiral by a rotation and a dilation of the previous edge in the control polygon of the curve, what the authors name *typical curve*. Certain relations between the scaling factor and the rotation angle give rise to aesthetic Bézier spirals starting with any initial edge of the control polygon and for any degree of the Bézier curve.

Inspired by this work, in [3] Farin extends the method by considering Bézier curves whose control polygon is obtained by the action of a given matrix on the previous edge of the control polygon. His insight is to exploit the invariance of the curvature and torsion under subdivision to give some conditions on the matrix and its singular values that give rise to aesthetic Bézier curves for any initial edge, what he calls *Class A matrices* and *Class A Bézier curves*.

However, counterexamples to Farin's conditions have been produced (see [2] and [5]), that is, matrices for which these conditions hold but they do not generate curves with monotonic curvature. Moreover in [2], Cao and Wang give conditions on the eigenvalues of a (2×2 or 3×3) symmetric matrix that generates an aesthetic (planar or spatial) Bézier curve.

In this talk, we present a simple explicit formula for the curvature of planar Bézier curves generated by Farin's method. This formula is easily obtained by the invariance under subdivision property and from it there can be derived conditions on the eigenvalues of a general matrix and the initial edge of the control polygon that give rise to aesthetic Bézier curves. This approach gives a common framework to the previous works and recovers the results in [4] and [2] as particular cases. For more details we refer to [1].

Joint work with: Leonardo Fernández-Jambrina, María Jesús Vázquez-Gallo.

References

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